

1. *Big-Oh my!* You have been asked to choose one of two possible algorithms. You are given just two facts. First, one algorithm has an asymptotic running time of  $O(n^{1.1})$  and the other  $O(n \log_2 n)$ . Secondly,  $n$  will be really huge. Which would you choose and why?
2. *Probability.* Suppose you roll a pair of 6-sided dice. What is the probability that the sum of pairs' values will be even?
3. *Calculus.* Consider the following function:

$$\Gamma(a) = \int_0^{\infty} t^{a-1} e^{-t} dt.$$

Show that  $\Gamma(a) = (a - 1)\Gamma(a - 1)$ .

4. *Algorithms and data structures.* Consider an undirected graph  $G = (V, E)$ , where  $V$  is the set of its vertices and  $E$  its edges. We say  $G$  is *bipartite* if there exists a partitioning of the vertices,  $V = V_1 \cup V_2$ , such that
  - a.  $V_1 \cap V_2 = \emptyset$ , that is,  $V_1$  and  $V_2$  are disjoint; and
  - b. for every edge  $(u, v) \in E$ ,  $u \in V_1$  and  $v \in V_2$  or  $v \in V_1$  and  $u \in V_2$ . That is, all edges go between  $V_1$  and  $V_2$ , and no edges lie wholly within a single partition.

Give an algorithm to check whether a graph is bipartite.